Pick up your notes and a calculator from the front!

Notes: (Topics 1.13 – 1.14) Modeling Functions and Applications

t L_1 (age in weeks)	4	5	6	8	12
$W(t)$ L_2 (weight in kg)	4.2	4.4	4.8	5.1	5.7

To get Y_1: Alpha → Trace → 1

Example 1: The age (in weeks) and weight (in kilograms) of 5 randomly selected babies from a particular pediatrician's office are listed in the table above.

A linear regression $y = \boxed{a} + \boxed{b}x$ can be used to model these data, where y is the predicted weight of a baby (in kg) that is x weeks old.

$a = 3.545$ $b = 0.185$

a) Write the equation of the linear model for these data.

Linear Regression Model: $y = 3.545 + 0.185x$

b) Using the linear model from part a), what is the predicted weight (in kilograms) of a baby that is 10 weeks old?

$y_1(10) = 5.395$ kg

c) The weight of a sixth baby is 5.3 kg. Using the model from part a), what is the age (in weeks) of this baby?

$$y = \begin{cases} y_1 = 3.545 + 0.185x \\ y_2 = 5.3 \end{cases}$$

2nd trace 5 Enter 3xs

$x = 9.4864\ldots$ weeks

Selecting an Appropriate Model Type

While some problems will indicate which model should be used, you will also be expected to select an appropriate model type based on a given table of values or by the context of the problem.

Linear Models: roughly constant rates of change

Quadratic Models: roughly linear rates of change or roughly symmetric with a single maximum/minimum or context involving area

Cubic Models: context involving volume

Example 2: For each of the following situations, determine whether a linear, quadratic, or cubic model would be most appropriate.

a) Balloons are filled with water in preparation for an epic water balloon battle. Each water balloon is roughly spherical The radius of each water balloon is measured relative to the amount of water it holds.

cubic

b) Totino's pizzas are on sale at the local grocery store. The price of one pizza varies between \$1.99 - \$2.19 depending on the variety. The total number of Totino's pizzas are counted relative to the total price of the purchase.

linear

c) A sprinkler is placed in a yard to water the grass. The sprinkler rotates in a circular pattern and waters all the grass between the sprinkler head and the furthest distance it reaches. The radius of the circular path is measured relative to the area watered by the sprinkler.

quadratic

x	0	0.4	0.9	1.2	1.7	2.2	2.9	3.4
y	5	10.6	15.4	17.1	18.0	16.5	10.2	2.8

center

Example 3: The table above provides data for 8 ordered pairs (x, y).

a) Which function type best models the data in the table: linear, quadratic, or cubic? Explain your answer using characteristics from the data in the table.

quadratic

b) Write the equation of the regression model for the data in the table $y = ax^2 + bx + c$

$y = -4.883\ldots x^2 + 15.958\ldots x + 4.995\ldots$

Residuals

Residual = Actual Value − Predicted Value

Residual = $y - \hat{y}$

comes from regression equation

t (age in weeks)	4	5	6	8	12
W (weight in kg)	4.2	4.4	4.8	5.1	5.7

Example 4: Using the model from **Example** 1, what is the residual of the baby that is 5 weeks old? Interpret the meaning of this value in the context of this problem.

$y_1(5)$ or $3.585+0.185$

Actual Value: 4.4

Predicted Value: 4.47

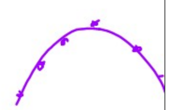
Residual: $4.4 - 4.47 = -0.07$

Interpretation: model overestimated the weights

Pick up your practice sheet and a calculator from the front! Get started, we will go over it BEFORE the end of class.

Worksheet A: (Topics 1.13 – 1.14) Modeling and Applications

t (in seconds)	0.1	0.5	0.9	1.5	1.9	2.3	2.6
$H(t)$ (in meters)	1.4	5.7	8.4	9.6	8.4	5.6	2.5



a) Based on this situation and the data presented in the table, would a linear, quadratic, or cubic function be most appropriate to model this data? Give a reason for your answer.

quadratic

b) Find the appropriate regression function to model these data.

$y = -4.915x^2 + 13.714x + 0.07$

c) Using the model found in part b, what is the predicted height of the football, in meters, at time $t = 1.3$ seconds?

$y_1(1.3) = 9.592$ meters

t (months)	0	1	13.5	19	26.5	31.5	35
$A(t)$ (in $)	87.58	124.15	164.61	185.97	152.60	122.42	90.98

a) Based on the data presented in the table, would a linear, quadratic, or cubic function be most appropriate to model this data? Give a reason for your answer.

quadratic

b) Find the appropriate regression function to model these data.

$y = -0.269x^2 + 9.162x + 99.972$

c) Using the model found in part b, what is the predicted price of one share of Amazon stock when $t = 16$ months (April 6, 2021)?

$y_1(16) = \$177.79$

d) The actual price for one share of Amazon stock on April 6, 2021 ($t = 16$) was $\boxed{\$168.61}$. What is the residual for this value?

actual value − pred val

$168.61 - 177.79 = -\$9.18$

t (years)	2018 0	2019 1	2020 2	2021 3	2023 5
$R(t)$ (in %)	1.5	2.5	1.9	0.5	4.5

Cubic

a) The data in the table can be modeled by the cubic regression function $y = ax^3 + bx^2 + cx + d$. Write the equation of this cubic regression function.

$0.293..x^3 + -1.899...x^2 + 2.759..x + 1.467..$

b) Based on the model found in part a, what is the predicted Federal Funds Rate for the year $\boxed{2026}$ ($t = 8$)?

$Y_1(8) = 52.357...\%$

c) The highest Federal Fund Rate in US history was 20% in 1980. Based on this information and the answer found in part b, do you think the cubic regression model found in part a is useful in predicting rates into the future? Explain your reasoning.

no; model was not a good predictor beyond historical rate

t (years)	0	10	22	30	37
$N(t)$ (in quadrillion BTUs)	12.4	21.8	20.4	19.3	22.6

a) The data in the table can be modeled by the cubic regression function $y = ax^3 + bx^2 + cx + d$. Write the equation of this cubic regression function.

$y = 0.00156x^3 - 0.0978x^2 + 1.762x + 12.40$

b) Based on the model found in part a, what was the predicted natural gas consumption for the US, in quadrillions of BTUs, for the year 1967 ($t = 7$)?

$y_1(7) = 20.475$ BTUs

Pick up your notes and a calculator from the front!

(Topic 2.1) Change in Arithmetic and Geometric Sequences

A sequence is a function from the whole numbers to the real numbers.

This means that we are only able to "plug" in whole numbers (0, 1, 2, 3, …) into a sequence but we can get any real number as the output.

As a result, when we graph a sequence, we will have points but we cannot "connect" them together to form a line or curve.

Example 1: Consider the sequence defined by $a_n = 4n - 3$. Find a_1 and a_7.

$a_1 = 4(1) - 3 = 1$

$a_7 = 4(7) - 3 = 25$

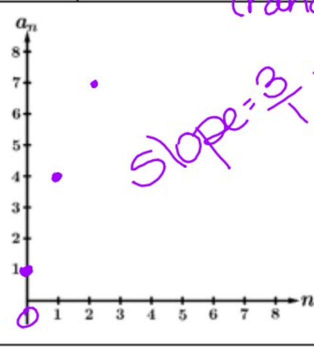
In this course, we will study two important types of sequences: <u>**arithmetic sequences**</u> and <u>**geometric sequences**</u>.

Arithmetic Sequences

Property of Successive Terms	Formulas/Equations	Notes
Successive terms have a **common difference**, or constant rate of change. add/subtract by the same # ex: 1, 3, 5, 7	$a_n = a_0 + dn$ **or** $a_n = a_k + d(n-k)$ where $a_0 =$ initial value $d =$ common difference $a_k =$ k^{th} term of sequence (random term given)	Arithmetic sequences behave like **linear functions**, except they are not continuous. Increasing arithmetic sequences increase equally each step. (slope always stays the same!)

Example

$a_n = \boxed{3}n + 1$ ← common difference



slope $= \frac{3}{1} = 3$

n	a_n
0	1
1	4
2	7
3	10

+3

$d = 3$

Example 2: For each of the following, determine if the sequence could be arithmetic. If yes, identify the common difference.

a) $s_n = n^2 - 3$

not arithmetic

b) $s_n = 6 - 2n$

yes; d = -2

c) $-7, -2, 3, 8, 13, \ldots$

+5 +5 +5

yes; d = 5

d) $1, -2, 3, -4, 5, \ldots$

not arithmetic

Example 3: Let a_n be an arithmetic sequence with $\boxed{a_3 = 8}$ and $d = -3$. Find an expression for a_n, and use the expression to find a_{12}.

$a_n = a_k + d(n-k)$

$a_k = a_3 = 8$

$k = 3$

$a_n = a_3 - 3(n-3)$

$= 8 - 3n + 9$

$a_n = 17 - 3n$

$a_{12} = 17 - 3(12)$

$a_{12} = -19$

Example 4: Let a_n be an arithmetic sequence with $\boxed{a_2 = 7}$ and $a_6 = 9$. Find an expression for a_n, and use the expression to find a_{24}.

$a_6 = a_2 + d(6-2)$

$9 = 7 + 4d$

$-7 \quad -7$

$\frac{2}{4} = \frac{4d}{4}$

$1/2 = d$

more things

$a_n = 7 + \frac{1}{2}(n-2)$

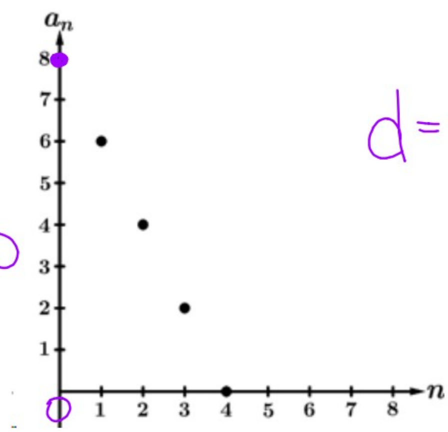
$= 7 + \frac{1}{2}n - 1$

combine like terms

$a_n = 6 + \frac{1}{2}n$

$a_{24} = 6 + \frac{1}{2}(24)$

$a_{24} = 18$



8, 6, 4, 2, 0

$d = -2$

$a_0 = 8$

Example 5: Several terms of the arithmetic sequence a_n are shown above. Find an expression for a_n and use the expression to find a_{17}.

$a_n = a_0 + dn$

$a_n = 8 - 2n$

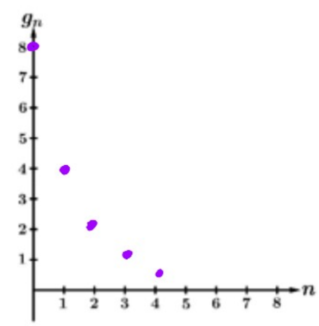
$a_{17} = 8 - 2(17)$

$a_{17} = -26$

Geometric Sequences		
Property of Successive Terms	**Formulas/Equations**	**Notes**
Successive terms have a **common ratio**, or constant proportional change. multiply/$\underbrace{\text{divide}}_{\text{written as a fraction}}$ by the same #	$g_n = g_0 r^n$ **or** $g_n = g_k r^{(n-k)} \rightarrow g_k(r)^{n-k}$ where $g_0 =$ initial value $r =$ common ratio $g_k =$ k^{th} term	Geometric sequences behave like **exponential functions**, except they are not continuous. {exponent is a variable} Increasing geometric sequences increase by a larger amount each step. (% increase always stays the same!)

Example

$g_n = 8\left(\frac{1}{2}\right)^n$ ← common ratio



n	g_n
0	8
1	4
2	2
3	1
4	½

$\div 2 \rightarrow r = \frac{1}{2}$

Example 6: For each of the following, determine if the sequence could be geometric. If yes, identify the common ratio.

a) $s_n = 3n^2$

not geometric

b) $s_n = 4(2)^{n-1}$

yes; r = 2

c) 1, 3, 2, 6, 4, 12, 8, 24, ...

not geometric

d) 16, − 8, 4, − 2, 1, ...

÷ −2

yes; r = −½

Example 7: Let g_n be a geometric sequence with $\boxed{g_1 = 12}$ and $r = 2$. Find an expression for g_n, and use the expression to find g_4.

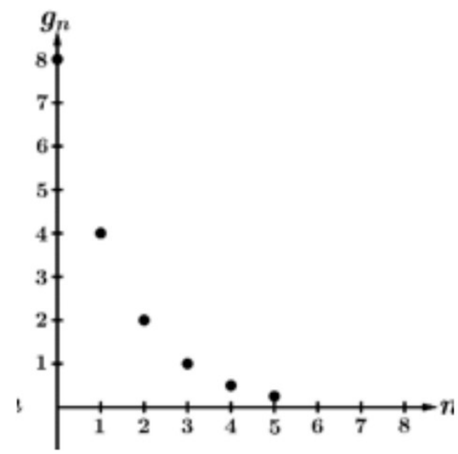
$g_n = g_k(r)^{n-k}$

k^{th} term

$g_n = 12(2)^{n-1}$

$g_4 = 12(2)^{4-1}$

$g_4 = 96$



Example 8: Several terms of the geometric sequence g_n are shown above. Find an expression for g_n and use the expression to find g_{10}.

n	g_n
0	8
1	4
2	2
3	1
4	$\frac{1}{2}$

given in table

used already x

$g_1 = 4$

$r = \frac{1}{2}$

$g_n = 4\left(\frac{1}{2}\right)^{n-1}$

$8\left(\frac{1}{2}\right)^{10} = \frac{1}{128}$

$g_{10} = 4\left(\frac{1}{2}\right)^{10-1}$

$g_{10} = \frac{1}{128}$

Worksheet A: (Topic 2.1) Arithmetic and Geometric Sequences

Directions: For each of the following, determine if the given sequence is arithmetic, geometric, or neither.

1. $12, 7, 2, -3, -8, \ldots$

2. $5, 10, 20, 40, \ldots$

3. $20, 10, 5, \frac{5}{2}, \ldots$

4. $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, 3, \ldots$

5. $1, 1, 2, 3, 5, 8, 13, \ldots$

6. $b_n = \frac{n+3}{2}$

arithmetic

$d = \frac{1}{2}$

n	b_n
0	$3/2$
1	$4/2 = 2$
2	$5/2$
3	$6/2 = 3$

Directions: Let a_n be an arithmetic sequence with the following properties. For each of the following, find an expression for a_n, and then find a_{11}. $a_n = a_0 + dn \quad a_n = a_k + d(n-k)$

↳ big #s ↳ lil #s

7. $a_3 = 7$ and $a_8 = 17$

8. $\boxed{a_2 = -3}$ and $a_6 = -9$

$-9 = -3 + d(\underbrace{6-2}_{4})$

$-9 = \cancel{-3} + 4d$

$+3 \quad \cancel{+3}$

$\frac{-6}{4} = \frac{\cancel{4}d}{\cancel{4}}$

$\boxed{-3/2 = d}$

$a_n = -3 - \frac{3}{2}(n$

$= -3 - \frac{3}{2}n$

$a_n = -\frac{3}{2}n$

$a_{11} =$

9. $a_5 = 7$ and $d = -4$

$a_n = 7 - 4(n-5)$

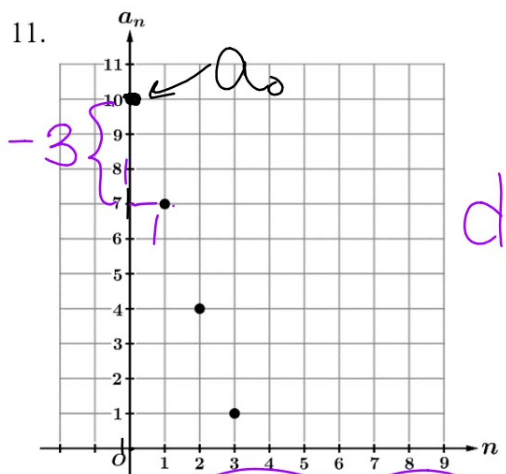
$= 7 - 4n + 20$

$a_n = 27 - 4n$

$a_{11} = 27 - 4(11) = -17$

10. $a_4 = -1$ and $d = \frac{2}{3}$

11.



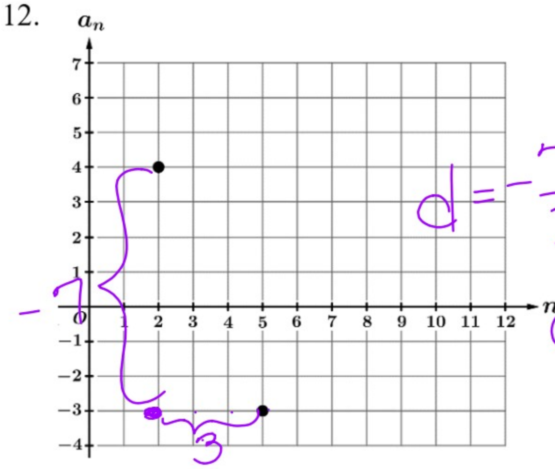
$d = \frac{-3}{1} = -3$

$a_n = 10 - 3n$

$a_{11} = 10 - 3(11)$

$a_{11} = -23$

12.



$d = -\frac{7}{3}$

$a_K(y)$

$4 - \frac{7}{3}$

$a_n = a_0 + dn$

$a_n = a_K + d(n-K)$

Directions: Let g_n be a geometric sequence with the following properties. For each of the following, find an expression for g_n, and then find g_6.

$g_n = g_0 r^n$ $\quad g_n = g_k(r)^{n-k}$

13. $g_1 = 5$ and $r = -2$

14. $g_2 = 8$ and $r = \frac{1}{2}$

15. $g_2 = 1$ and $g_5 = 27$

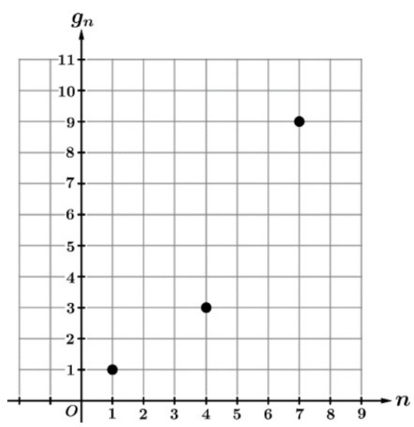
$27 = 1(r)^{5-2}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

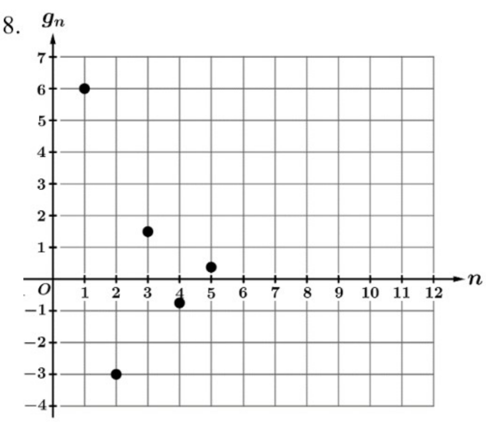
$3 = r$

16. $g_4 = -12$ and $g_7 = \frac{32}{9}$

17.


g_n
11
10
9
8
7
6
5
4
3
2
1
O
1 2 3 4 5 6 7 8 9
n

18.


g_n
7
6
5
4
3
2
1
O
1 2 3 4 5 6 7 8 9 10 11 12
n
−1
−2
−3
−4

Pick up your notes and a calculator from the front!

Notes: (Topic 2.2) Change in Linear and Exponential Functions

In the last section, we learned that arithmetic sequences behave like linear functions and that geometric sequences behave like exponential functions.

We will look more closely at these connections today.

Arithmetic Sequences and Linear Functions	
Arithmetic Sequences	**Linear Functions**
$a_n = a_0 + dn$	$f(x) = b + mx$ **Slope-Intercept Form**
$a_n = a_k + d(n - k)$	$f(x) = y_i + m(x - x_i)$ **Point-Slope Form**

Geometric Sequences and Exponential Functions	
Geometric Sequences	**Exponential Functions**
$g_n = g_0 r^n$	$f(x) = ab^x$ or $f(x) = ar^x$
$g_n = g_k r^{(n-k)}$	$f(x) = y_i r^{(x-x_i)}$

Linear Functions vs. Exponential Functions	
Linear Functions	**Exponential Functions**
$f(x) = b + mx$	$f(x) = ab^x$
Over equal-length input-value intervals, the output values change at a **constant rate**.	Over equal-length input-value intervals, the output values change **proportionately**.
The change (m) in y is based on **addition**.	The change (b) in y is based on **multiplication**.
If you have **two points**, you can write the equation of a linear function, exponential function, arithmetic sequence, or a geometric sequence.	

Example 1: Selected values of several functions are given in the table below. For each table, determine if the function could be linear, exponential, or neither. Give a reason for your answer.

dd/sub by same # ↳ mult./divide by same #

a)

x	$f(x)$
0	7
3	5
6	3
9	1
12	-1

-2

linear

b)

x	$g(x)$
1	0
2	1
3	4
4	9
5	16

$+1$, $+3$, $+5$, $+7$

neither

c)

x	$h(x)$
0	1
2	2
4	4
6	8
8	16

$\times 2$

exponential

d)

x	$k(x)$
5	80
10	40
15	20
20	10
25	5

$\div$

exponential

Example 2: A wild rumor is spreading that Mr. Passwater won 3rd place in the World's Strongest Man Contest (Mr. Passwater ~~definitely~~ probably didn't start the rumor). The number of people that have heard the rumor can be modeled using a geometric sequence, where the 43 people had heard the rumor on day 3 and 140 people have heard the rumor on day 6. According to the model, how many people, to the nearest whole number, have heard the rumor by day 10?

$g_3 = 43$

$\boxed{g_6 = 140}$

Two Ran

① Find r

② Write y

(pick a

③ Plug in

for

$140 = 43r^{6-3}$

$\frac{140}{43} = \frac{\cancel{43}r^3}{\cancel{43}}$

$\sqrt[3]{\frac{140}{43}} = \sqrt[3]{r^3}$

$\sqrt[3]{\frac{140}{43}}$ or $\left(\frac{140}{43}\right)^{1/3} = r$

$g_n = 140\left(\sqrt[3]{\frac{140}{43}}\right)^{n-6}$

$g_{10} = 140\left(\sqrt[3]{\frac{140}{43}}\right)^{10-6}$

$g_{10} = 675.57\ldots$

676 people that heard the rumor by the 10th day

Example 3: A large theater has rows of seats where the number of seats in each row can be modeled by an arithmetic sequence. If the fifth row has 31 seats and the eleventh row has 49 seats, determine how many seats there are in the twenty-fifth row.

dn

d(n-k)

$\boxed{a_5 = 31}$ $a_{11} = 49$

$49 = 31 + d(11-5)$

$49 = \cancel{31} + 6d$

$-31 \quad -\cancel{31}$

$\frac{18}{6} = \frac{\cancel{6}d}{\cancel{6}}$

$3 = d$

$a_n = 31 + 3(n-5)$

$a_{25} = 31 + 3(25-5)$

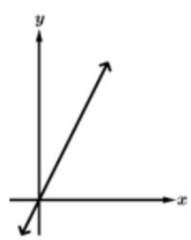
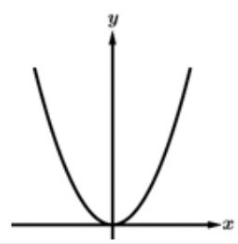
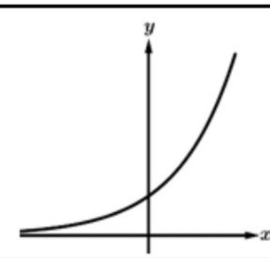
$a_{25} = 91$

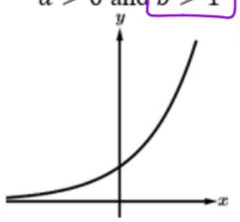
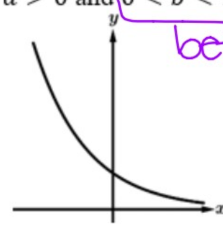
91 seats in 25^{th} row

Pick up your notes and a calculator from the front!

Notes: (Topic 2.3) Exponential Functions

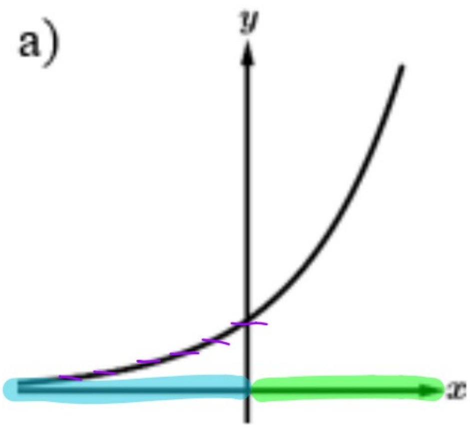
In math, we study several different types of functions. Three types of functions seem to keep showing up in every math course from Algebra 1 to Calculus. These three families of functions are also featured predominantly on the SAT and ACT!

Function	$f(x) = 2x$	$g(x) = x^2$	$h(x) = 2^x$
Name	Linear	Quadratic	Exponential
Graph			

Key Characteristics of Exponential Functions	
An exponential function has the general form $f(x) = a(b)^x, \quad b > 0$ where a and b are constants with $a \neq 0$ and $b \neq 1$.	a represents the initial amount. b represents the base/common ratio.
Exponential Growth $a > 0$ and $\boxed{b > 1}$ greater than 1 	**Exponential Decay** $a > 0$ and $\boxed{0 < b < 1}$ between 0 & 1 
Increasing vs. Decreasing Exponential functions are **always increasing** or **always decreasing**! They will never switch from one to the other, so they have **no relative (local) extrema**.	**Concave Up vs. Concave Down** Exponential functions are **always concave up** or **always concave down**! They will never switch concavity, so they have **no points of inflection**.
End Behavior For exponential functions in general form, as the input values (x) increase/decrease without bound, the output values (y) will increase/decrease without bound or they will approach zero.	**End Behavior Limit Statements** $\lim_{x\to+\infty} ab^x = \boxed{\infty}$ or $\lim_{x\to+\infty} ab^x = \boxed{-\infty}$ or $\lim_{x\to+\infty} ab^x = \boxed{0}$ Only Options As Answers

Example 1: Write limit statements for the end behavior of the following exponential functions.

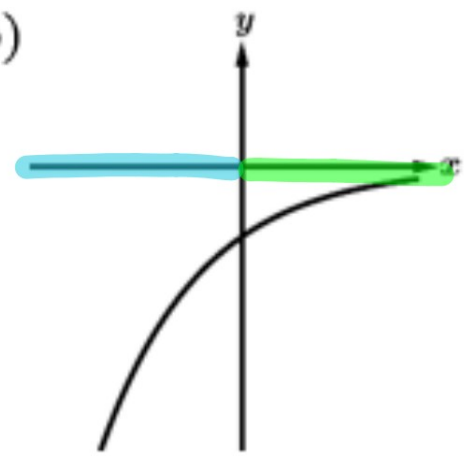
a)



Left: $\lim_{x\to-\infty} f(x) = 0$

Right: $\lim_{x\to\infty} f(x) = \infty$

b)



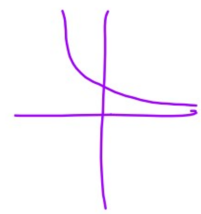
Left: $\lim_{x\to-\infty} f(x) = -\infty$

Right: $\lim_{x\to\infty} f(x) = 0$

c) $g(x) = 5\left(\dfrac{2}{3}\right.$

b

deca

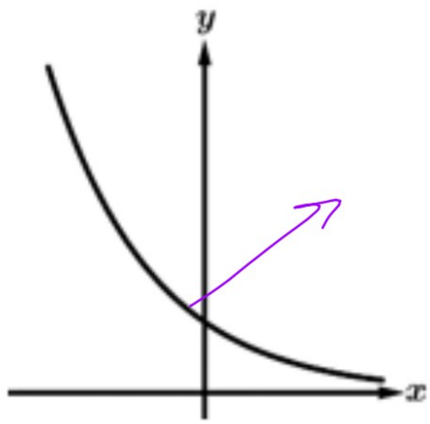


Left: $\lim_{x\to-\infty} g(x) =$

Right: $\lim_{x\to\infty} g(x) =$

Example 2: For each of the following, determine if the exponential function is increasing/decreasing and concave up/down.

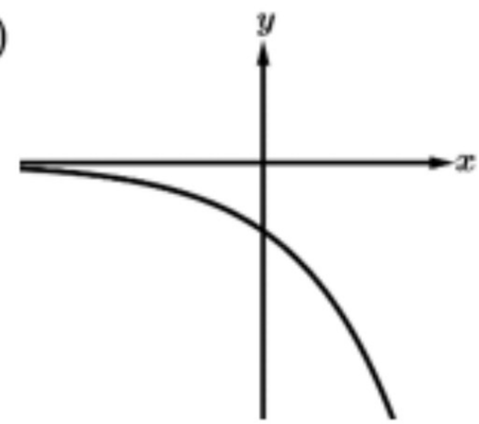
a)



Concave Up or Concave Down

Increasing or Decreasing

b)

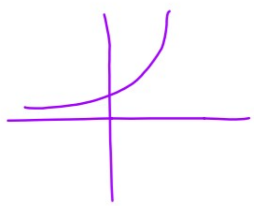


Concave Up or Concave Down

Increasing or Decreasing

c) $h(x) = 3(4)^x$

growth



Concave Up or Concave Down

Increasing or Decreasing

Section 2.2

Directions: Determine if the function is linear, exponential or neither.

1)

x	f(x)
0	-22
1	-15
2	-8
3	-1
4	6

+7

Linear

2)

x	g(x)
3	1
5	2
7	4
9	8
11	16

·2

Exponential

Neither

3)

x	h(x)
2	1
4	2
6	6
8	24
10	120

×2
×3
×4
×5

Directions: Given the following terms, find either the common difference or ratio. Then find the term indicated.

4) $a_{12} = 77;\ a_{34} = 209$

Find a_8

$a_n = a_0 + dn$

$a_n = a_k + d(n-k)$

$a_{32} = a_{10} + d(\_\_\_)$

5) $\boxed{a_{10} = -59};\ a_{32} = -257$

Find a_{21}

$-257 = -59 + d(\overbrace{32-10}^{22})$

$-257 = -\cancel{59} + 22d$

$+59 \quad +\cancel{59}$

$\dfrac{-198}{22} = \dfrac{\cancel{22}d}{\cancel{22}}$

$\boxed{-9 = d}$

$a_n = -59 -$

$a_{21} = -59$

$a_{21} = -15$

6) $g_3 = -25;\ g_6 = -3125$

Find g_8

$g_n = g_0 r^n$

$g_n = g_k r^{n-k}$

7) $\boxed{g_2 = 16};\ g_5 = -1024$

Find g_1

$\dfrac{-1024}{16} = \dfrac{\cancel{16}r^{\overset{5-2}{3}}}{\cancel{16}}$

$\sqrt[3]{-64} = \sqrt[3]{r^3}$

$\boxed{r = -4}$

$g_n = 16(-4)$

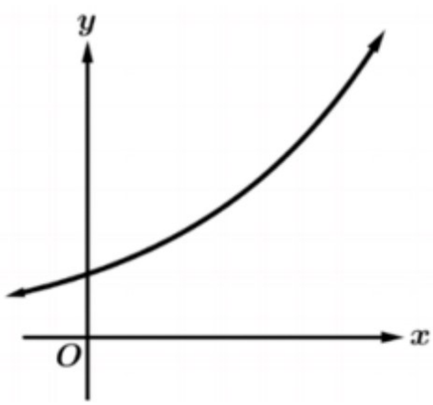
$g_1 = 16(-4)$

$g_1 = -4$

Section 2.3

Directions: Write *limit statements* for the following functions to describe its end behavior. Then determine its concavity and whether it increases or decreases.

8)



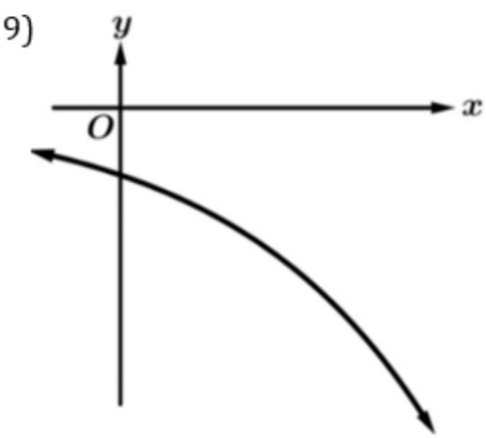
Left: $\lim_{x \to -\infty} f(x) = 0$

Right: $\lim_{x \to \infty} f(x) = \infty$

Concave up or Concave down

Increasing or Decreasing

9)



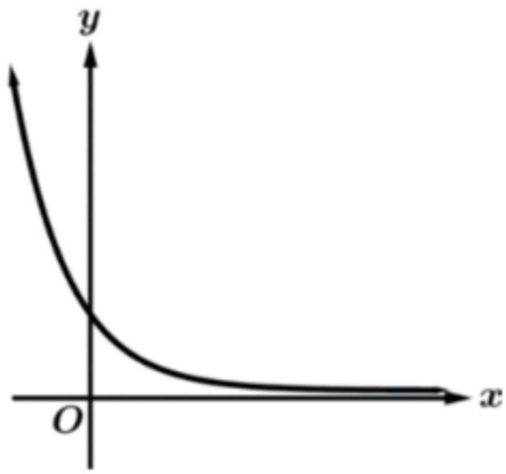
Left: $\lim_{x \to -\infty} f(x) = 0$

Right: $\lim_{x \to \infty} f(x) = -\infty$

Concave up or Concave down

Increasing or Decreasing

10)



Left: $\lim_{x \to -\infty} f(x) = \infty$

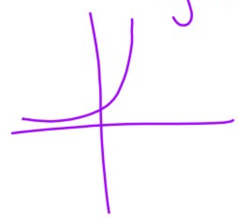
Right: $\lim_{x \to \infty} f(x) = 0$

Concave up or Concave down

Increasing or Decreasing

11) $f(x) = 2(3)^x$

growth



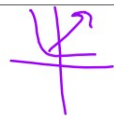
Left: $\lim_{x \to -\infty} f(x) = 0$

Right: $\lim_{x \to \infty} f(x) = \infty$

Concave up or Concave down

Increasing or Decreasing

12) $g(x) = 4\left(\frac{1}{2}\right)^x$



decay

Left: $\lim_{x \to -\infty} g(x) = \infty$

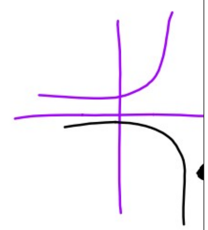
Right: $\lim_{x \to \infty} g(x) = 0$

Concave up or Concave down

Increasing or Decreasing

reflect over x-axis (flips)

13) $h(x) = -5\left(\frac{7}{3}\right)^x$



growth

Left: $\lim_{x \to -\infty} h(x) = 0$

Right: $\lim_{x \to \infty} h(x) = -\infty$

Concave up or Concave down

Increasing or Decreasing

$$a_n = a_k + d(n-k)$$

$$g_n = g_k r^{n-k}$$

Pick up your notes and a calculator from the front!

Notes: (Topic 2.4) Exponential Function Manipulation

Review of Important Exponent Rules		
Product Property	**Power Property**	**Negative Exponent Property**
$b^m b^n = b^{m+n}$	$\left(b^m\right)^n = b^{mn}$	$b^{-n} = \dfrac{1}{b^n}$
Examples: $x^5 x^3 = x^{5+3} = x^8$ $2^x 2^3 = 2^{x+3}$	**Examples:** $\left(x^5\right)^3 = x^{5(3)} = x^{15}$ $\left(2^x\right)^3 = 2^{3x}$	**Examples:** $x^{-3} = \dfrac{1}{x^3}$ $2^{-x} = \dfrac{1}{2^x}$

Example 1: Determine the horizontal transformations of each of the following exponential functions.

a) $f(x) = 4^{\boxed{x+2}}$ left 2

b) $g(x) = 2^{3x}$ dilation by factor of $\frac{1}{3}$

c) $h(x) = 9^{x/2}$ dilation by factor of 2

d) $k(x) = 5^{\boxed{x-1}}$ right 1

Now, let's reexamine the exponential function from **Example 1a**, and see if we can use our exponent properties to rearrange the function into an equivalent form.

We can use the **Product Property** (in reverse) to show that $4^{x+2} = 4^x 4^2 = 16(4^x)$.
(add/sub in exponent)

This means that $f(x) = 4^{x+2}$ is equivalent to writing $f(x) = 16(4)^x$.

When we rewrite the exponential function in this way, there is no long a horizontal translation, but now we have a vertical dilation by a factor of 16.

Example 2: Each of the following exponential functions has a horizontal translation. For each, write an equivalent representation that has a vertical dilation and no horizontal translation $(f(x) = ab^x)$.

a) $f(x) = 2^{x+3}$

$= 2^x 2^3$

$f(x) = 8(2^x)$

b) $g(x) = 3^{x-2}$

$= 3^x 3^{-2}$

$g(x) = \frac{1}{9}(3^x)$

c) $k(x) = 4(3)^{x+2}$

$= 4 \cdot 3^x 3^2$

$k(x) = 36(3^x)$

We can also use the **Power Property of Exponents** to show that every horizontal dilation of an exponential function,
(mult/divide)
$\left(f(x) = b^{cx}\right)$, is equivalent to changing the base of the exponential function $\left(f(x) = \left(b^{c}\right)^{x}\right)$.

Example 3: Which of the following functions is an equivalent form of the function $y = 9^{2x}$

(A) $f(x) = 3^{x}$

(9

(B) $f(x) = 3 \cdot 9^{x}$

8

(C) $f(x) = 18^{x}$

(D) $f(x) = 81^{x}$

Example 4: Which of the following functions is an equivalent form of the function $y = 9 \cdot 4^x$?

~~(A) $f(x) = 3 \cdot 16^{x/2}$~~

~~(B) $f(x) = 3 \cdot 16^{2x}$~~

$\sqrt{16}$

(C) $f(x) = 9 \cdot 16^{x/2} = 9(16^{1/2})^x$

(D) $f(x) = 9 \cdot 16^{2x} = 9(16^2)^x$

Worksheet A: (Topic 2.4) Exponential Function Manipulation

Directions: Rewrite each of the following exponential functions in the equivalent general form $y = ab^x$, where a and b are positive constants.

1. $f(x) = 7^{x+2}$

$7^x 7^2$

$49(7^x)$

$49 \cdot 7^x$

2. $g(x) = 5^{x-1}$

$5^x 5^{-1}$

$\frac{1}{5}(5^x)$

3. $h(x) = 2^{x+3}$

$2^x 2^3$

$8(2^x)$

4. $k(x) = 3^{x-3}$

$3^x 3^{-3}$

$\frac{1}{27}(3^x)$

5. $p(x) = 2(4)^{x-1}$

$2 \cdot 4^x \cdot 4^{-1}$

$\frac{1}{2}(4^x)$

6. $m(x) = 3^{2x}$

$(3^2)^x$

9^x

7. $r(x) = 4^{x/2}$

$\sqrt{4} = (4^{1/2})^x$

2^x

8. $n(x) = 8^{x/3}$

$\sqrt[3]{8} = (8^{1/3})^x$

2^x

9. $s(x) = 5(2)^{3x}$

$5(2^3)^x$

$5(8^x) = 5(8$

10. Which of the following functions is an equivalent form of the function $f(x) = 4 \cdot 36^x$?

(A) $f(x) = 2 \cdot 6^{(x/2)}$

(B) $f(x) = 2 \cdot 6^{(2x)}$

(C) $f(x) = 4 \cdot 6^{(x/2)}$

(D) $f(x) = 4 \cdot 6^{(2x)}$

11. Which of the following functions is an equivalent form of the function $g(x) = 5 \cdot 3^{2x}$?

(A) $g(x) = 45^x$

(B) $g(x) = 5 \cdot 9^x$

(C) $g(x) = 25 \cdot 3^x$

(D) $g(x) = 25 \cdot 9^x$

12. The function h is given by $h(x) = 9 \cdot 4^{(x/2)}$. Which of the following is an equivalent form for $h(x)$?

$(4^{1/2})^x = \sqrt{4}$

~~(A)~~ $h(x) = 6 \cdot 2^x$

(B) $h(x) = 9 \cdot 2^x$

~~(C)~~ $h(x) = 18 \cdot 2^x$

(D) $h(x) = 9 \cdot 16^x$

13. The function k is given by $k(x) = a^2 \cdot 4^x$, where a is a positive constant. Which of the following is an equivalent form for $k(x)$?

~~(A)~~ $k(x) = a \cdot 2^{(x/2)}$

(B) $k(x) = a^2 \cdot 2^{(x/2)} \rightarrow (2^{1/2})^x$

~~(C)~~ $k(x) = a \cdot 16^{(x/2)}$

(D) $k(x) = a^2 \cdot 16^{(x/2)} \rightarrow (16^{1/2})^x$

14. Which of the following functions is an equivalent form of the function $p(x) = 3^{-2x}$?

(A) $p(x) = -(9)^x$

(B) $p(x) = (-9)^x$

(C) $p(x) = -\left(\frac{1}{9}\right)^x$

(D) $p(x) = \left(\frac{1}{9}\right)^x$

15. The function m is given by $m(x) = 8 \cdot 9^{(x/3)}$. Which of the following is an equivalent form for $m(x)$?

$\left(9^{1/3}\right)^x$

$\left(\sqrt[3]{9}\right)^x$

~~(A)~~ $m(x) = 2 \cdot 3^x$

~~(B)~~ $m(x) = 2 \cdot \left(\sqrt[3]{9}\right)^x$

(C) $m(x) = 8 \cdot 3^x$

(D) $m(x) = 8 \cdot \left(\sqrt[3]{9}\right)^x$

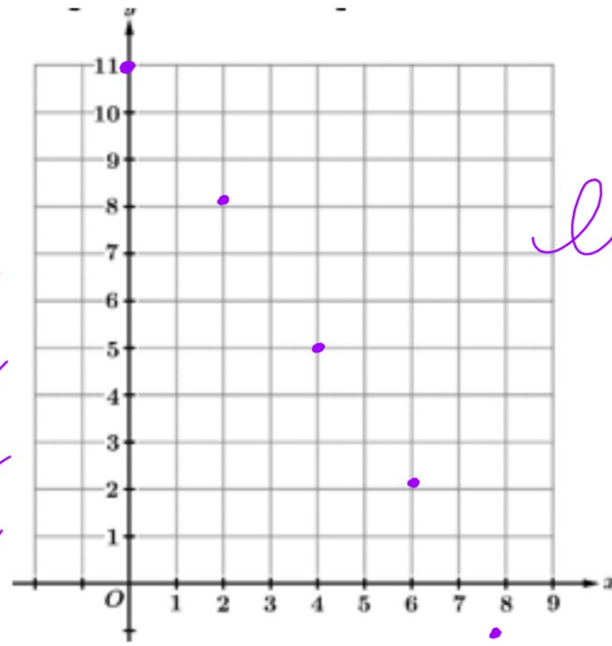
Pick up your notes and a calculator from the front!

Notes: (Topic 2.6) Competing Function Model Validation

Example 1: Selected values from several functions are given in the tables below. Sketch the scatterplot for each table. Then determine if a linear, quadratic, or exponential model is most appropriate.

a)

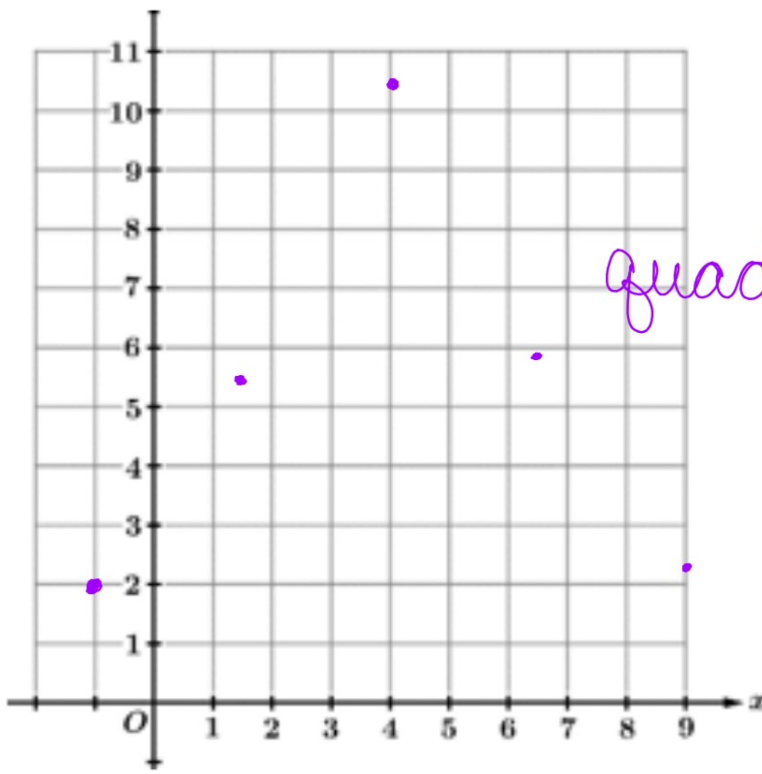
x	$f(x)$
0	11 ✓
2	8.2 ✓
4	5 ✓
6	2.3 ✓
8	−1 ✓



linear

b)

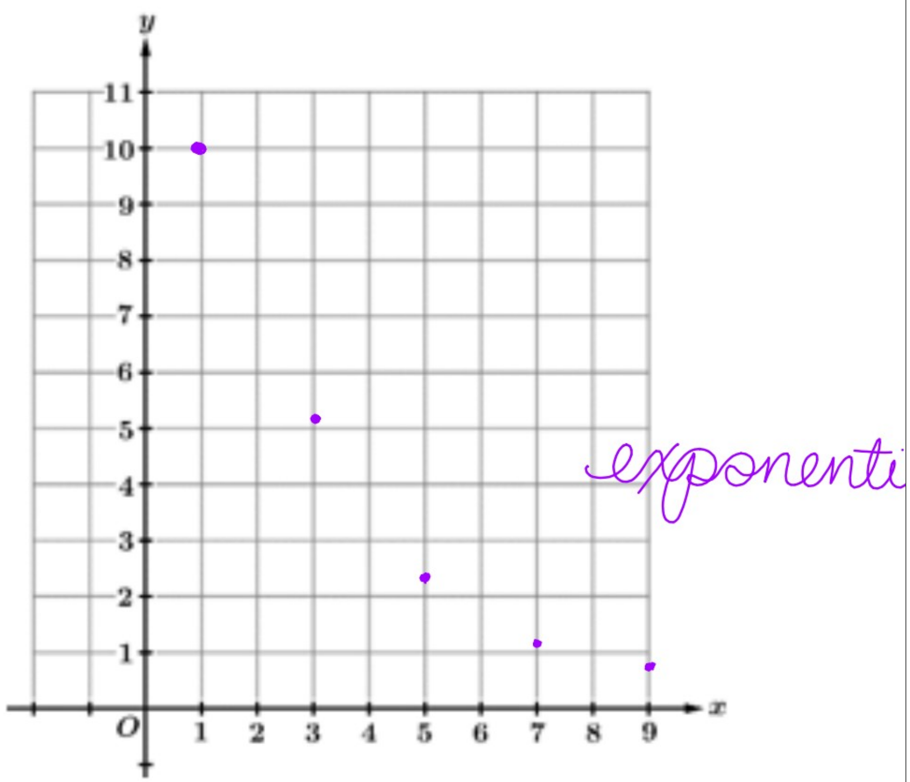
x	$g(x)$
-1	2
1.5	5.5
4	10.5
6.5	5.75
9	2.25



Quadratic

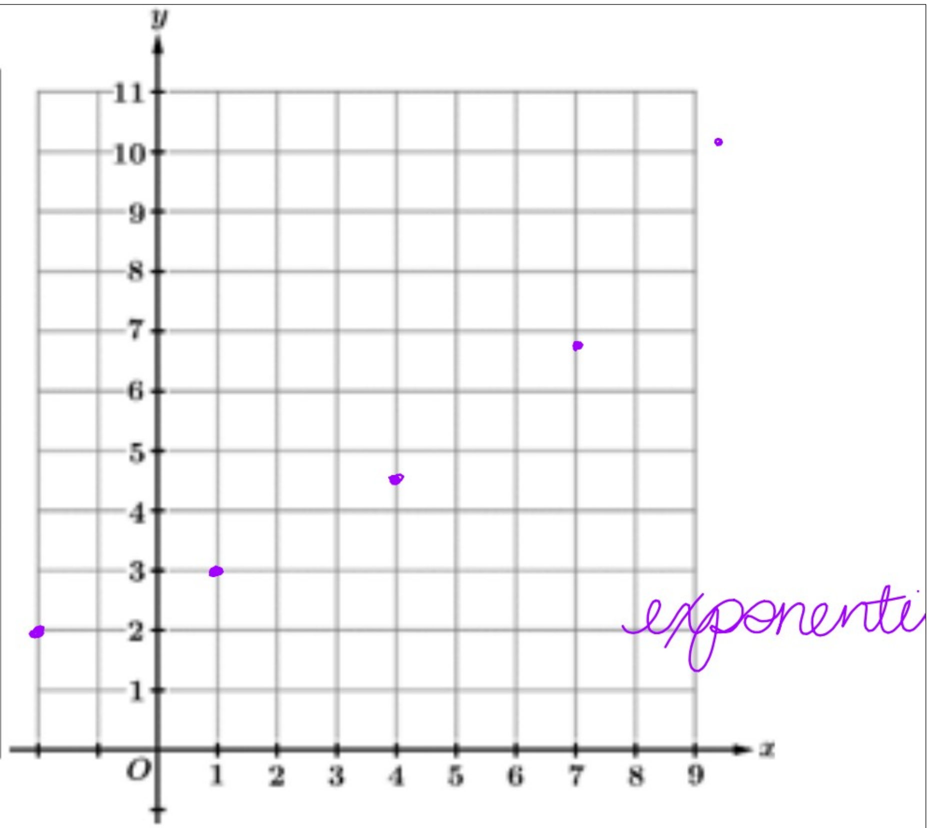
c)

x	$h(x)$
1	10
3	5.2
5	2.4
7	1.3
9	0.7



d)

x	$k(x)$
-2	2
1	3
4	4.5
7	6.75
10	10.25



Residual = Actual Output Value – Predicted Output Value

Example 2: The weight of newborn babies can be modeled by a linear function for the first four months after birth. Selected values for the weight $W(t)$, in kilograms, of a particular newborn baby are given in the table above, where t represents the number of months since birth.

a) Use the regression capabilities on your calculator to find a linear model of the form $y = a + bx$ for the weight (in kg) of this particular baby x months after birth. To Store: Alpha → Trace → 1

$y = 3.34 + 0.8x$

b) Use the model found in part a to predict the weight (in kg) of this baby 2.5 months after birth.

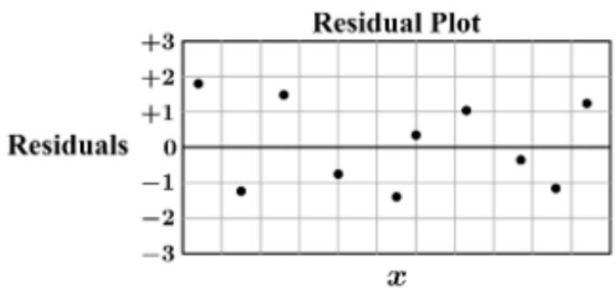
$y_1(2.5) = 5.34$ kg

c) The actual weight of this baby 2.5 months after birth was 5.5 kilograms. What is the residual for this weight? Did our model underestimate or overestimate the weight of this baby 2.5 months after birth?

$5.5 - 5.34 = 0.16$; underestimated

If a model for a given set of data is appropriate, the residual plot should appear without pattern.

If we can see a clear pattern in a residual plot, then the model used for the regression was not appropriate!



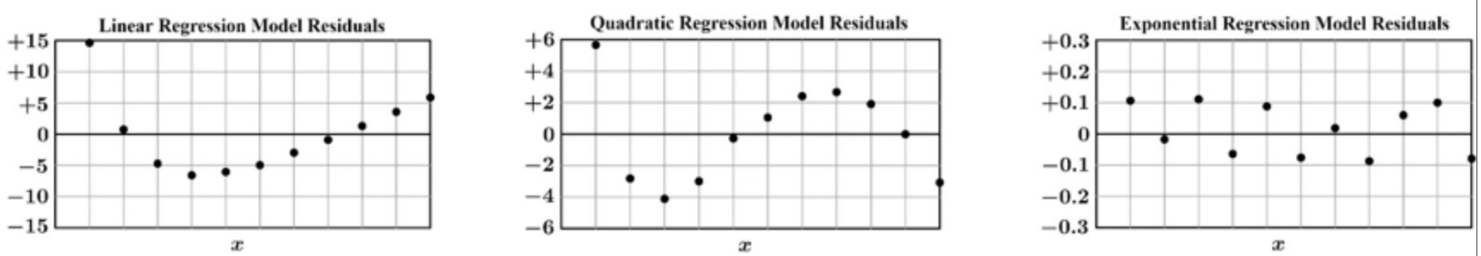
Example 3: An exponential regression was used to model a data set. The residual plot for the exponential regression model is shown above. Which of the following is the best conclusion about the appropriateness of the exponential regression model based on the corresponding residual plot?

(A) The exponential model is not appropriate because the residuals show no pattern.

(B) The exponential model is not appropriate because the residuals show a pattern.

(C) The exponential model is appropriate because the residuals show no pattern.

(D) The exponential model is appropriate because the residuals show a pattern.



Example 4: A group of AP Precalculus students used a set of data to create linear, quadratic, and exponential regression models. After creating the three models, the students created a residual plot for each model type (see above). Based on the three residual plots above, which model was most appropriate for the data? Give a reason for your answer based on the residual plots above.

exponential model b/c it shows
NO PATTERN!

Example 5: Mr. Passwater hopes to make it into the Guiness Book of World Records by painting the world's biggest mural on the sides of several downtown buildings. His mural will consist of many painted circles of various sizes, where each circle is painted a different color. He wants to create a model to determine how much paint is needed (in quarts) for a circle of radius r (in feet).

a) Should Mr. Passwater use a linear, quadratic, or exponential model in this situation? Explain your reasoning.

quadratic; area of the circle (paint)

b) After creating his model, Mr. Passwater uses it to purchase different special paint colors to use on his mural. In this situation do you think it is more appropriate for the model to underestimate or overestimate the actual amount of paint needed? Give a reason for your answer.

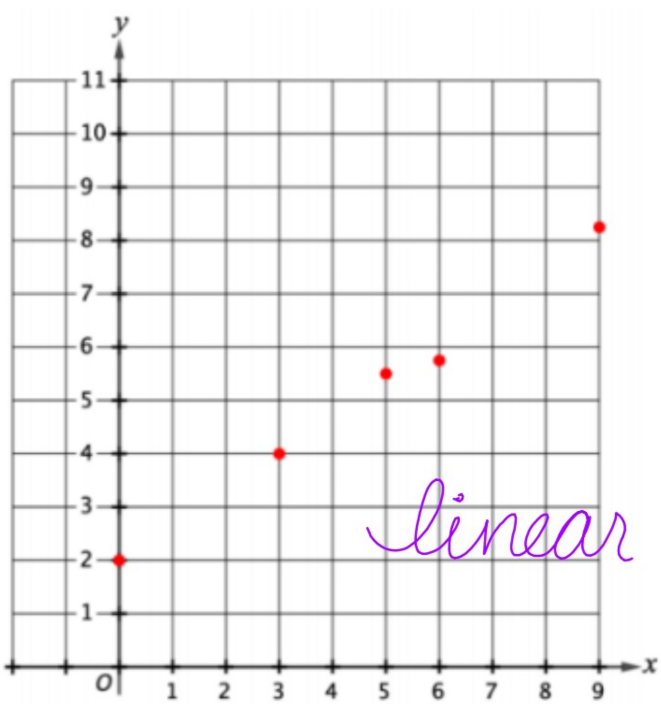
overestimate

Pick up your worksheet and a calculator from the front and get to work!

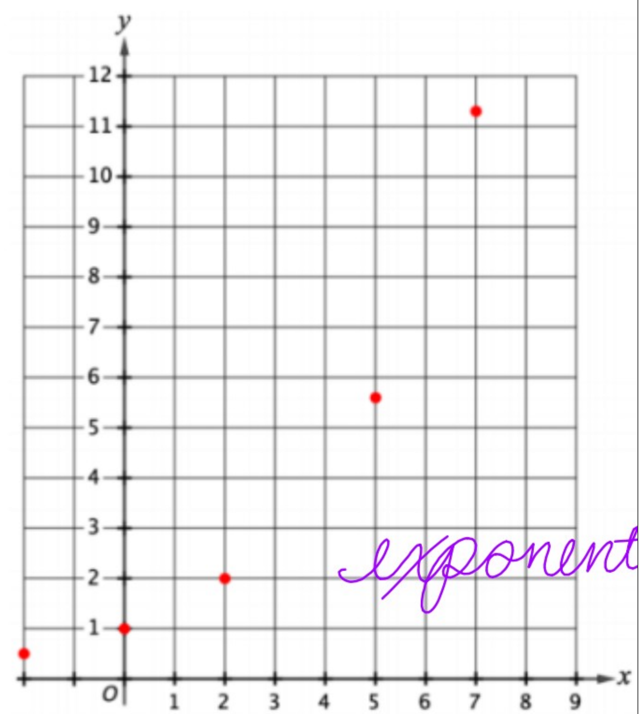
Worksheet A: (Topic 2.6) Competing Function Model Validation

Directions: Selected values from several functions are given in the tables below. Sketch the scatterplot for each table. Then determine if a linear, quadratic, or exponential model is most appropriate.

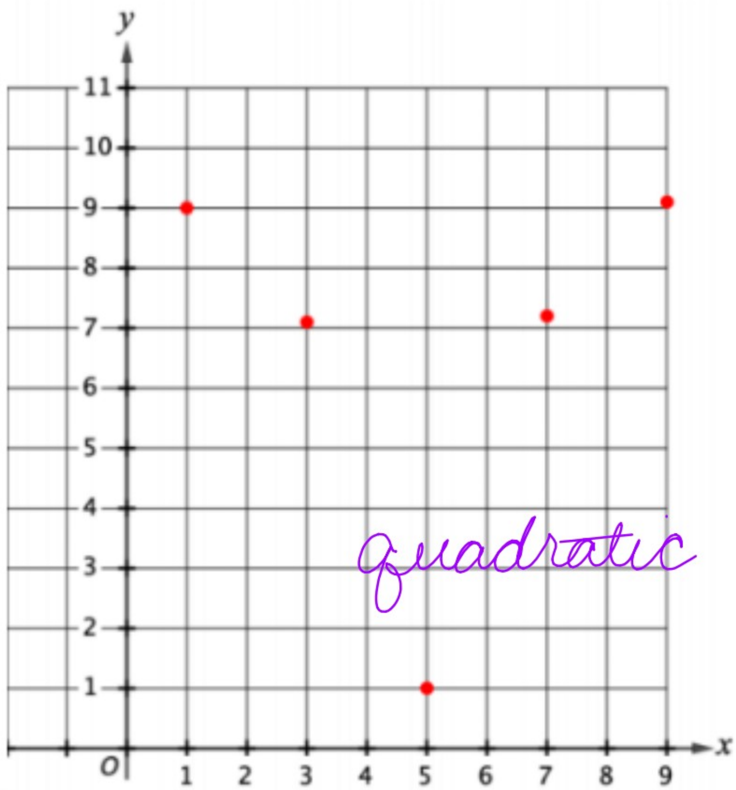
1)



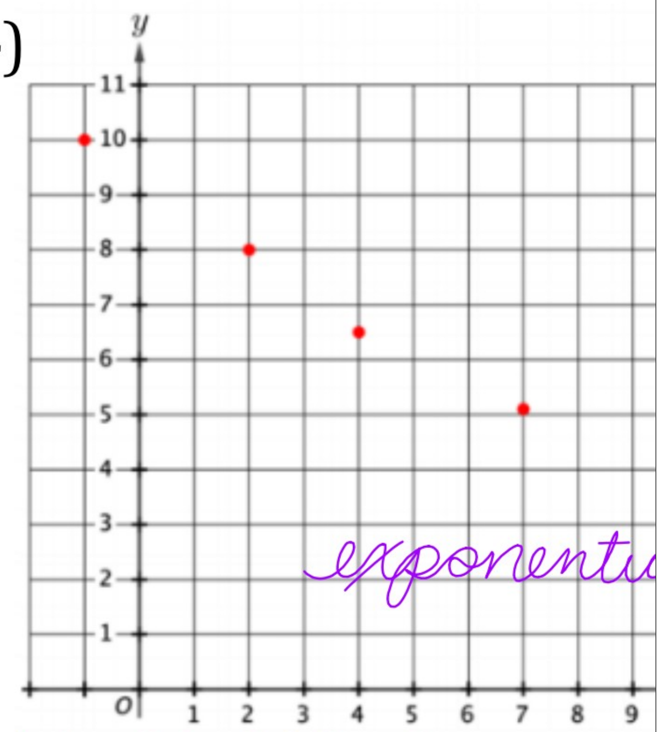
2)

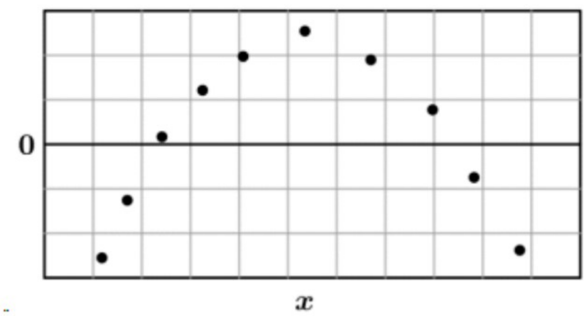


3)



4)





5. A quadratic regression was used to model a data set. The residual plot for the model is above. Which of the following statements about the appropriateness of the model is correct?

(A) A quadratic regression model is appropriate because the residuals do not show a linear pattern.
(B) A quadratic regression model is appropriate because the residuals show a quadratic pattern.
(C) A quadratic regression model is not appropriate because the residuals do not show a linear pattern.
(D) A quadratic regression model is not appropriate because the residuals show a quadratic pattern.

Years Since 1950	0	10	20	30	40	50	60	70
Total World Population (in billions)	2.5	3.02	3.7	4.44	5.32	6.15	6.99	7.84

actual

6. Over the years 1950 – 2020, the total world population can be modeled by a linear function. Selected values for the total world population P, in billions, are given in the table above, where t represents the number of years since 1950

a) Use the regression capabilities on your calculator to find a linear model of the form $y = a + bx$ for the world population (in billions) x years since 1950.

$y = 2.2675 + 0.0779x$

b) According to the model found in part a, what was the world population in 1979, the year Mr. Passwater was born?

-1950

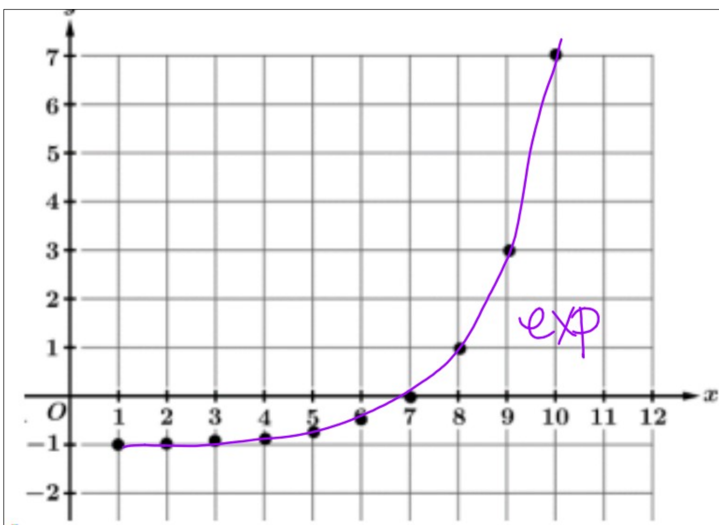
$y_1(29) = 4.5274$ billion

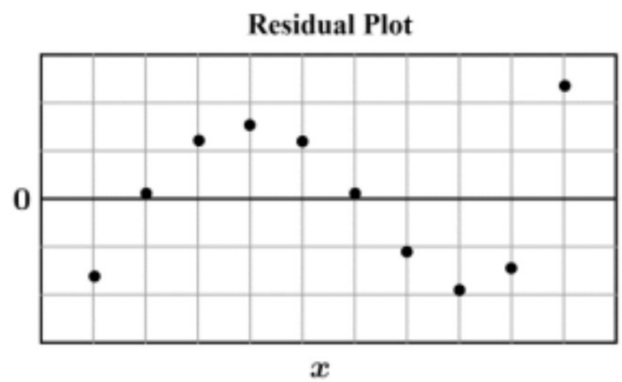
c) What is the residual of the total world population for the year 1990? Did our model underestimate or overestimate the total world population for the year1990?

(+) (−)

$y_1(40) = 5.3846$ billion (predicte

$5.32 - 5.38 = -0.06$; overestimate





7. A regression model was created for the data in the graph above (left). The residual plot for the model is given above (right). Which of the following statements about the regression model is best?

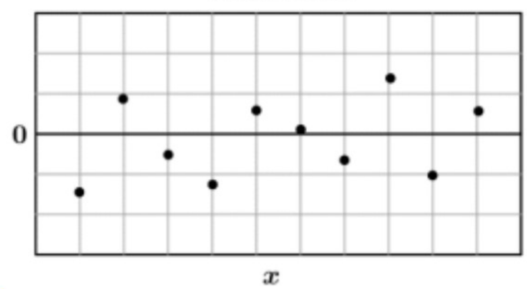
~~(A)~~ A quadratic regression model was used and the model is appropriate.

(B) A quadratic regression model was used and the model is not appropriate.

~~(C)~~ An exponential regression model was used and the model is appropriate.

(D) An exponential regression model was used and the model is not appropriate.

Residual Plot



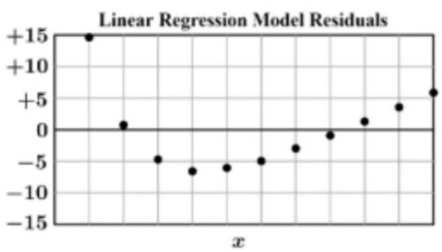
8. Mr. Passwater used a set of data to create a quadratic regression model. The residual plot for his model is shown above. Based on the residual plot above, which of the following conclusions is correct?

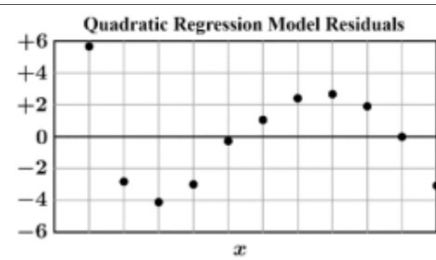
(A) The residual plot has no apparent pattern, so the quadratic model was appropriate. → No patter

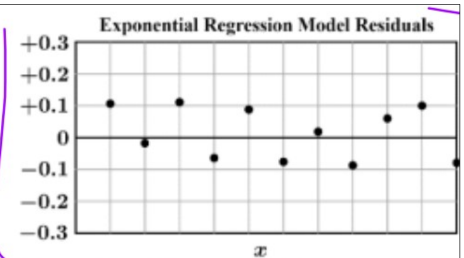
(B) The residual plot has no apparent pattern, so the quadratic model was not appropriate.

(A) The residual plot displays a pattern, so the quadratic model was appropriate.

(A) The residual plot displays a pattern, so the quadratic model was not appropriate.







9. A set of data was used to create a linear, a quadratic, and an exponential regression model. The residual plots for the three models are shown above. Based on the three residual plots, which of the following could be an appropriate model for the data?

linear | quadratic | exponential | logarithmic

(A) $y = 3 + 2x$ (B) $y = x^2 + 2x + 3$ (C) $y = 3(2)^x$ (D) $y = 3 + 2\log x$

10. Mr. Passwater loves to invest his money in mutual funds. Over the past twenty years, he has closely tracked how his account grows and noticed that each year his account grows by approximately 10.4%. If Mr. Passwater wants to find a function that models the amount of money in his account over time, should he use a linear, quadratic, or exponential model? Give a reason for your answer.

exponential

11. After Mr. Passwater creates his model from question 10, he uses the model to create a residual plot in order to check the appropriateness of his model. If his model was appropriate, what should he expect to see when looking at the residual plot?

NO PATTERN!!!!!!

NO PATTERN!